

The Nature of Prime Numbers as Rotational Transparency

*Interference in the C_4 group, the One-Third Theorem,
Crystalline Numbers and the proof by the leaps argument*

Abstract

We propose a **new geometric representation** of divisibility based on binary matrices and the C_4 symmetry group. The arithmetic properties of prime and composite numbers -- well known in themselves -- acquire in this representation a **visual manifestation** that reveals non-obvious aspects. The main results are: (1) the $N-1$ diagonal of the base matrix encodes the divisors of N (an elementary consequence, but geometrically significant); (2) the diagonal density $D\%$ in the C_4 overlay converges to $1/3$ (a new quantitative result); (3) the peripheral density satisfies $P\% \sim 6.17 \tau(N)/N^2$, linking geometry to the divisor function (new); (4) visual observation reveals **Crystalline Numbers** -- composites whose collisions fall exclusively on the diagonal and border -- and computational verification on 1000 numbers shows that **N is crystalline if and only if $N+1$ is prime** (for $N \geq 16$, zero exceptions). The proof relies on an elementary property (two numbers with prime sum are coprime), but its geometric manifestation is new and constitutes a geometric primality test.

Positioning of this work

The arithmetic properties of prime numbers have been studied for over two thousand years. This work does not claim to discover new facts about primality itself. Its contribution is threefold:

A new representation. The C_4 overlay matrix and the spatial distribution of collisions are original constructions not found in the existing literature.

New quantitative results. The One-Third Theorem ($D\% \rightarrow 1/3$), the formula $P\% \sim 6.17 \tau(N)/N^2$, and the Ratio D/P are quantities specific to this representation, not derivable from previous results.

A visual discovery path. Crystalline Numbers were identified by purely visual observation -- looking at collision maps -- and only subsequently linked to primality. The final proof relies on an elementary algebraic fact (coprimality of numbers with prime sum), but its geometric manifestation in the C_4 overlay is the original contribution.

This work belongs to the tradition of **visual and experimental mathematics**: new representations that make known properties visible and suggest new ones.

1. The Base Divisibility Matrix

The adopted matrix is a binary matrix of order $n \times n$ defined by:

$$A_{r,c} = 1 \text{ if and only if } r \text{ divides } c$$

The resulting structure is an upper triangle where the main diagonal ($r = c$) is entirely filled.

2. The N-1 Diagonal Theorem

Proposition. On the diagonal $r + c = N$ of the base matrix, cell $(r, N-r)$ contains value 1 if and only if r divides N .

Proof. By definition, $M_{r,c} = 1$ iff $r \mid c$. On the diagonal, $c = N - r$. The condition becomes $r \mid (N - r)$. Since $r \mid r$, we have $r \mid (N - r)$ iff $r \mid N$. QED

Corollary. N is prime if and only if the N-1 diagonal ($1 < r < N$) is empty.

*The proposition and its corollary are elementary consequences of the matrix definition. Their interest lies not in algebraic novelty, but in making primality a **geometrically visible** property: an empty diagonal in a matrix.*

3. The C_4 Group: Four Rotations

We define the total interference matrix as the sum of the base matrix with all its rotations in $C_4 = \{0, 90, 180, 270 \text{ degrees}\}$:

$$S(r, c) = M(r, c) + M_{90}(r, c) + M_{180}(r, c) + M_{270}(r, c)$$

Each cell takes values from 0 to 4. The matrix S is invariant under 90-degree rotation.

***Note.** The rotations add no information to divisor detection on the $N-1$ diagonal (Section 2). Their contribution is in generating an interference field whose spatial distribution distinguishes primes from composites.*

3.1 Exact result: cells with value 4

Observation (verified $N = 1..100$). For every odd N , exactly one cell has value 4 (the geometric center). For every even N , no cells with value 4 exist.

3.2 Visual comparison: $N = 30$ vs $N = 31$

N = 30 (2x3x5) -- OPAQUE

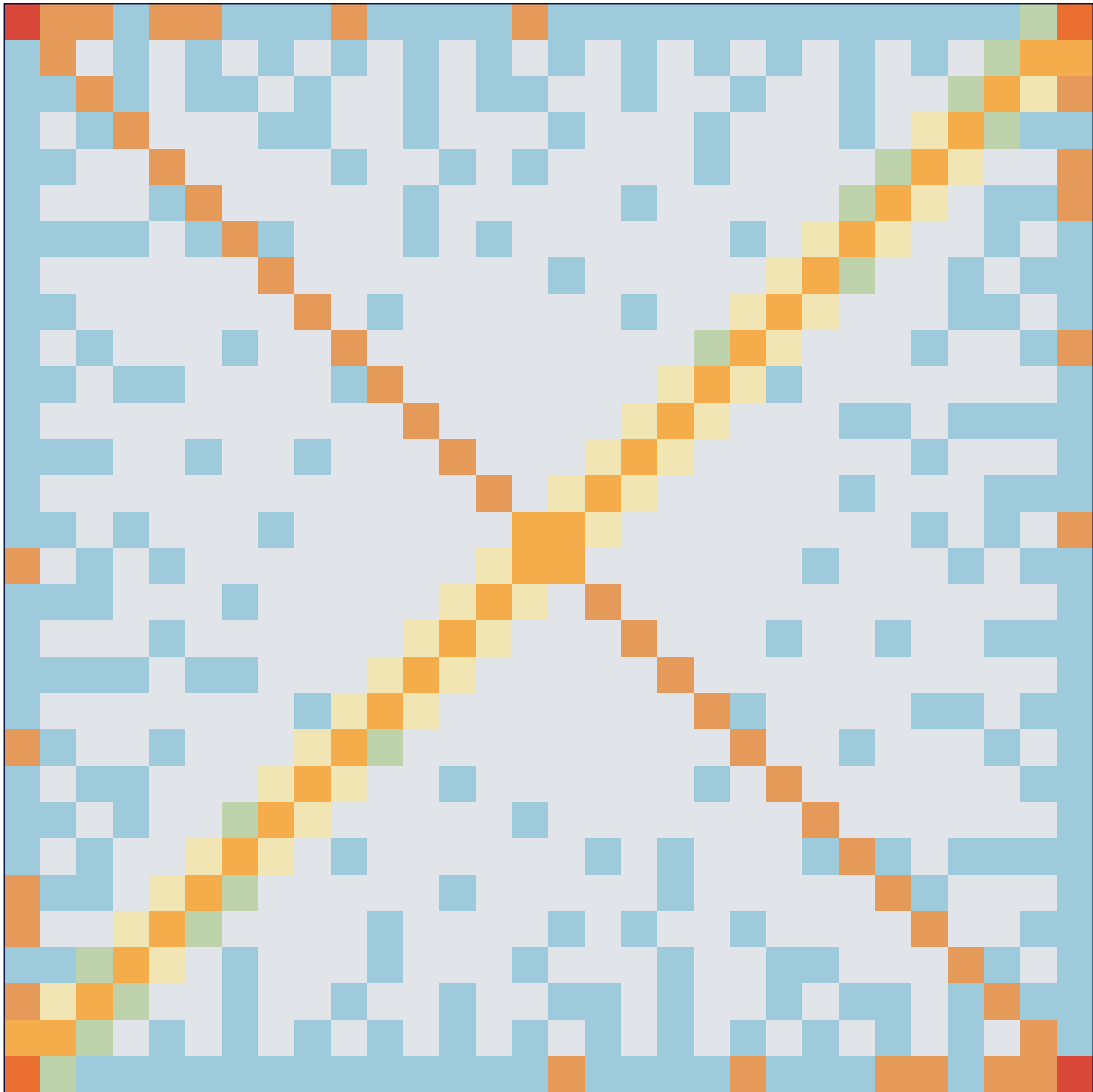


Fig. 1 -- N = 30. Diffuse collisions.

N = 31 (prime) -- TRANSPARENT

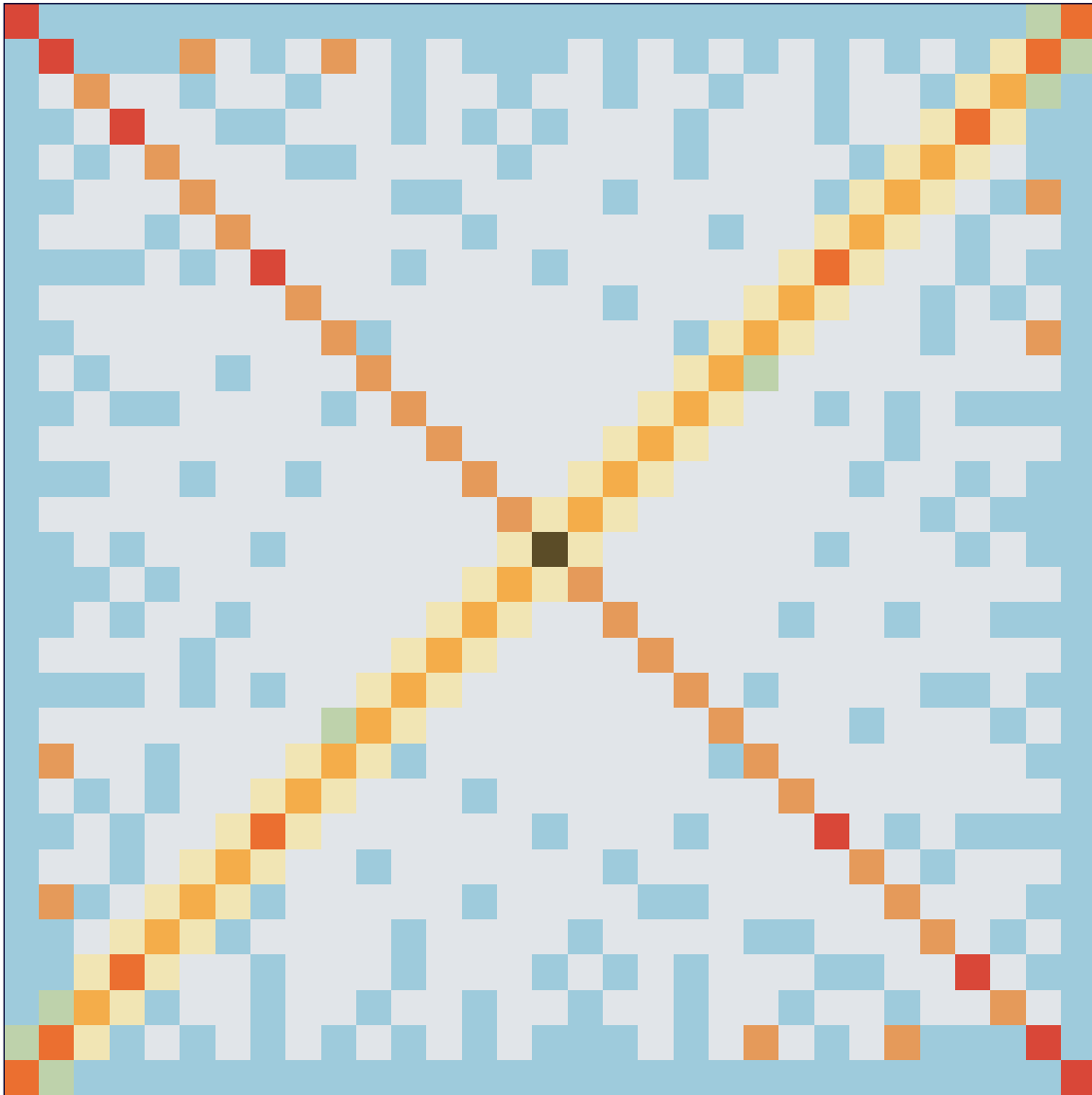


Fig. 2 -- N = 31. Collisions concentrated on diagonal, near-empty periphery.

4. Global Statistics and Their Limits

Global collision statistics lose discriminating power for large N : the prime/composite ratio converges to 1. This motivates looking at **where** collisions fall, not just how many.

5. Zonal Analysis: the Ratio D/P

We divide S into diagonal band (distance $\leq k$ from anti-diagonal) and periphery. Define:

$$D = (\text{cells} \geq 2 \text{ in band}) / (\text{cells in band}) ; P = \text{same for periphery} ; \\ \text{Ratio} = D/P$$

Ratio D/P -- green = primes, orange = composites

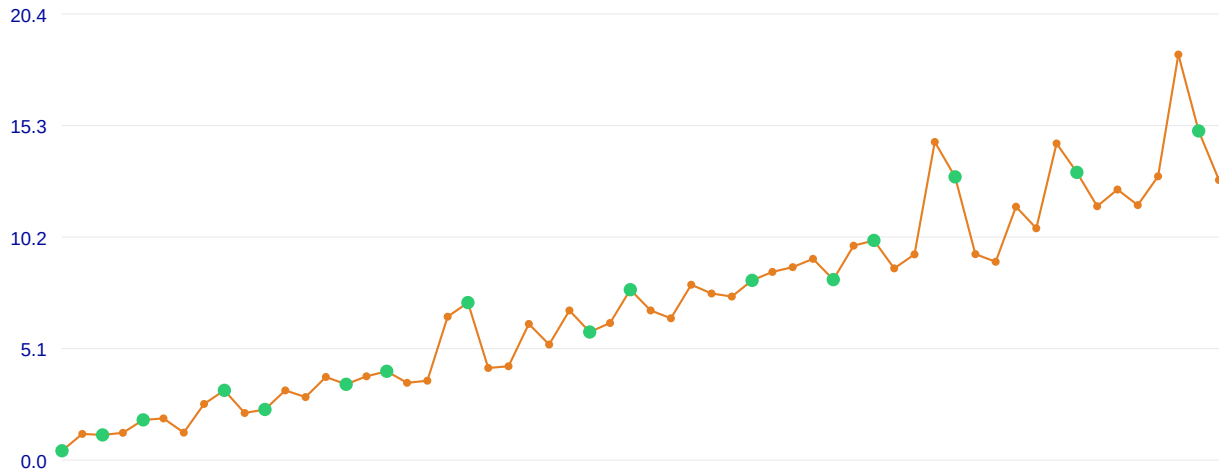


Fig. 3 -- Ratio D/P for N from 3 to 60.

6. Sensitivity to Band Width

The optimal band width is ± 1 : it produces the maximum gap between primes and composites.

7. The One-Third Theorem: $D\% \rightarrow 1/3$

Numerical analysis up to $N = 500$ reveals that $D\%$ converges to $1/3$. Deviation at $N = 500$: 0.047 percentage points.

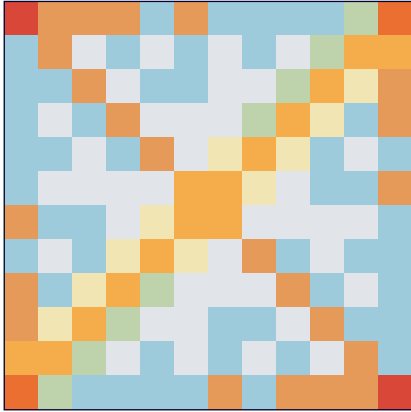
$$D\% - 1/3 \sim 0.26 / N \quad (r = 0.997)$$

The convergence follows from the structure of the ± 1 band: the main anti-diagonal is a guaranteed collision line (contributing $\sim N$ collisions out of $\sim 3N$ band cells).

8. The Factorization Signature

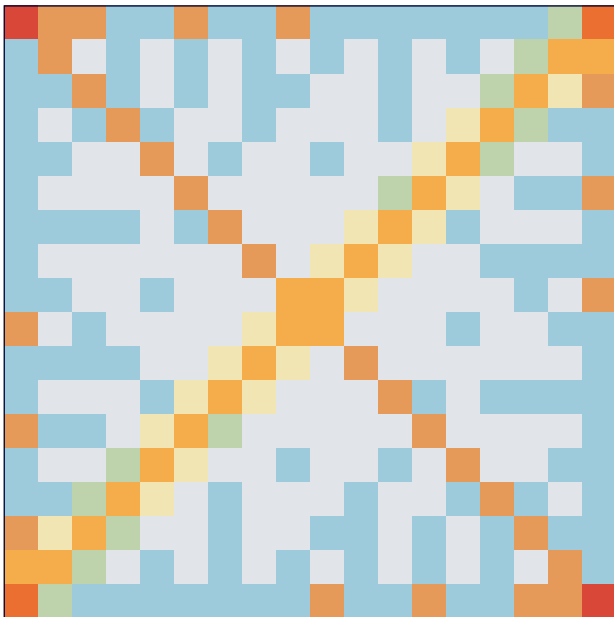
Comparing numbers sharing the same prime base reveals that the C_4 map encodes the factorization structure.

$N = 12$ ($2^2 \times 3$)

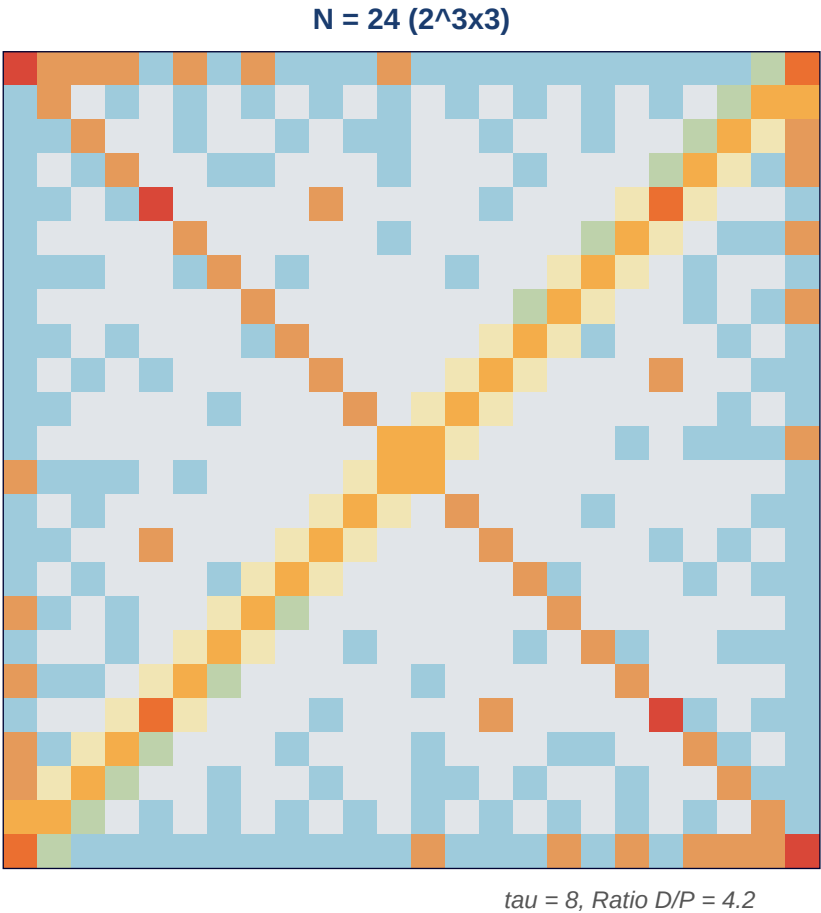


$\tau = 6$, Ratio $D/P = 2.2$

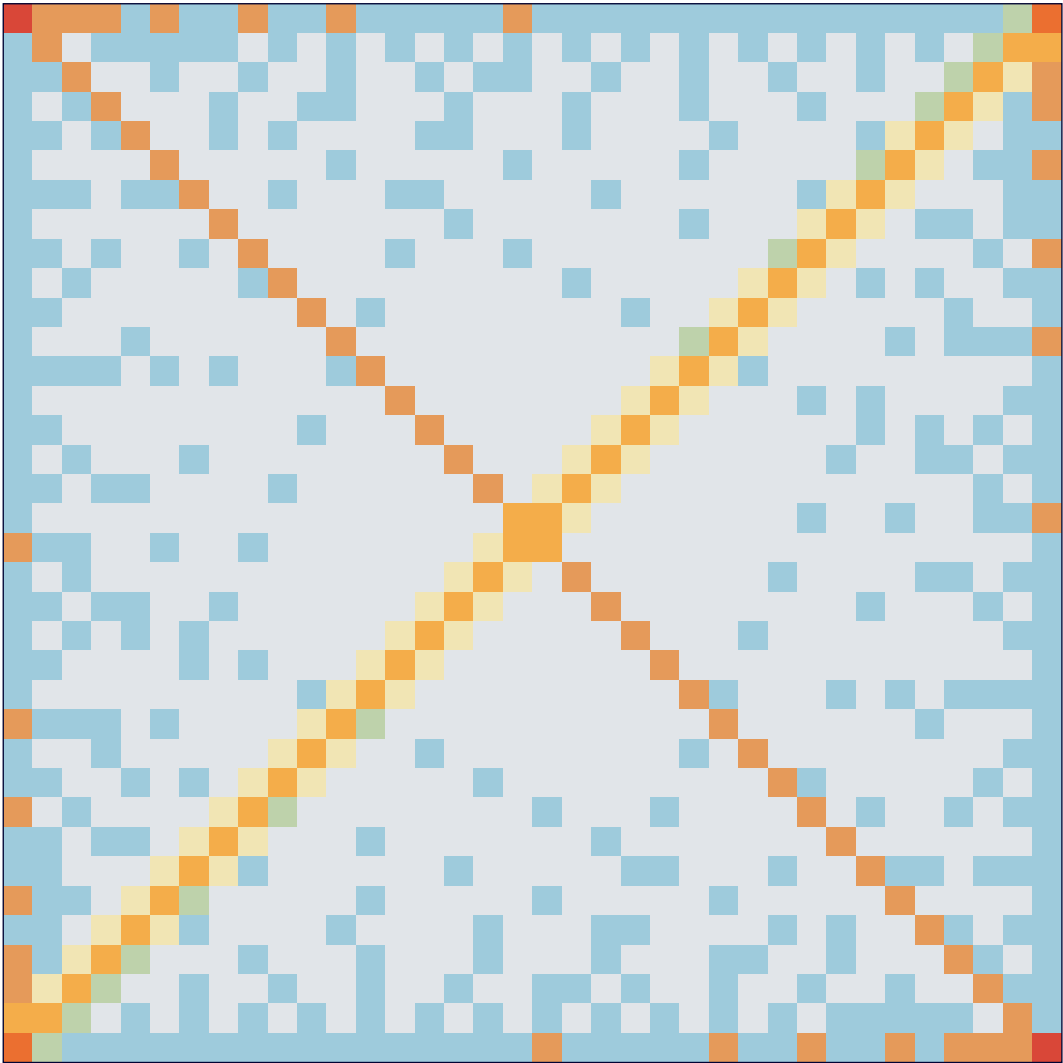
$N = 18$ (2×3^2)



$\tau = 6$, Ratio $D/P = 3.8$



$N = 36 (2^2 \times 3^2)$



$\tau = 9$, Ratio $D/P = 7.5$

9. Decay Laws and Analytic Connections

9.1 D% convergence: $D\% = 1/3 + 0.26/N$ ($r = 0.997$)

9.2 Peripheral decay: $P\% \sim c/N$, with $c \sim 1.51$ (primes) vs 1.86 (composites)

9.3 Ratio growth: $\text{Ratio } D/P \sim 0.053 N \ln(N)$

9.4 Main quantitative result:

$$P\%_{\text{composites}} \sim 6.17 \tau(N) / N^2 \quad (r = 0.964)$$

A geometric property of the C_4 overlay is directly proportional to a classical arithmetic function.

10. Crystalline Numbers

Visual analysis of collision maps revealed composites whose collisions fall exclusively on the X diagonal and border. We call them **Crystalline Numbers**.

10.1 Computational definition

A composite N is crystalline if every cell with value ≥ 2 in the C_4 overlay lies on the border (first/last row or column) or on the exact X diagonals ($r + c = N-1$ or $r = c$).

10.2 The discovery: Crystalline = $p - 1$

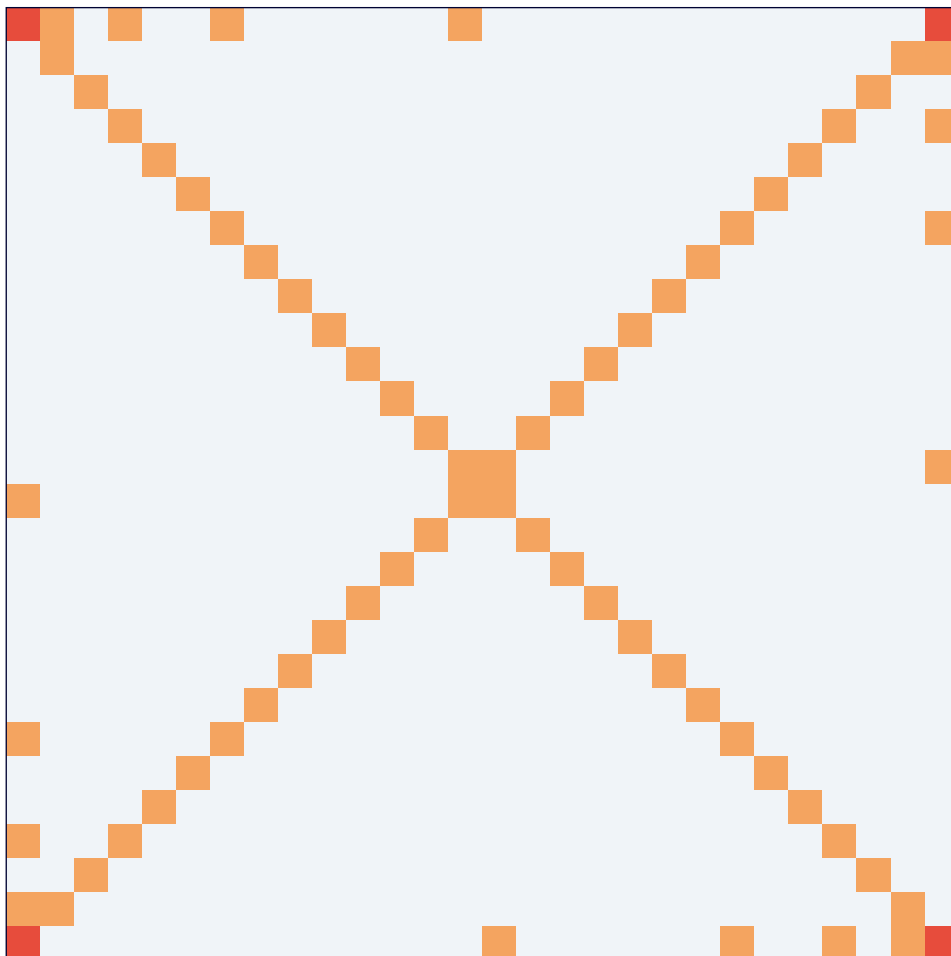
Visual observation produced a list. The list turned out to match $p-1$ numbers exactly.

For $N \geq 16$: N is crystalline if and only if $N + 1$ is prime.

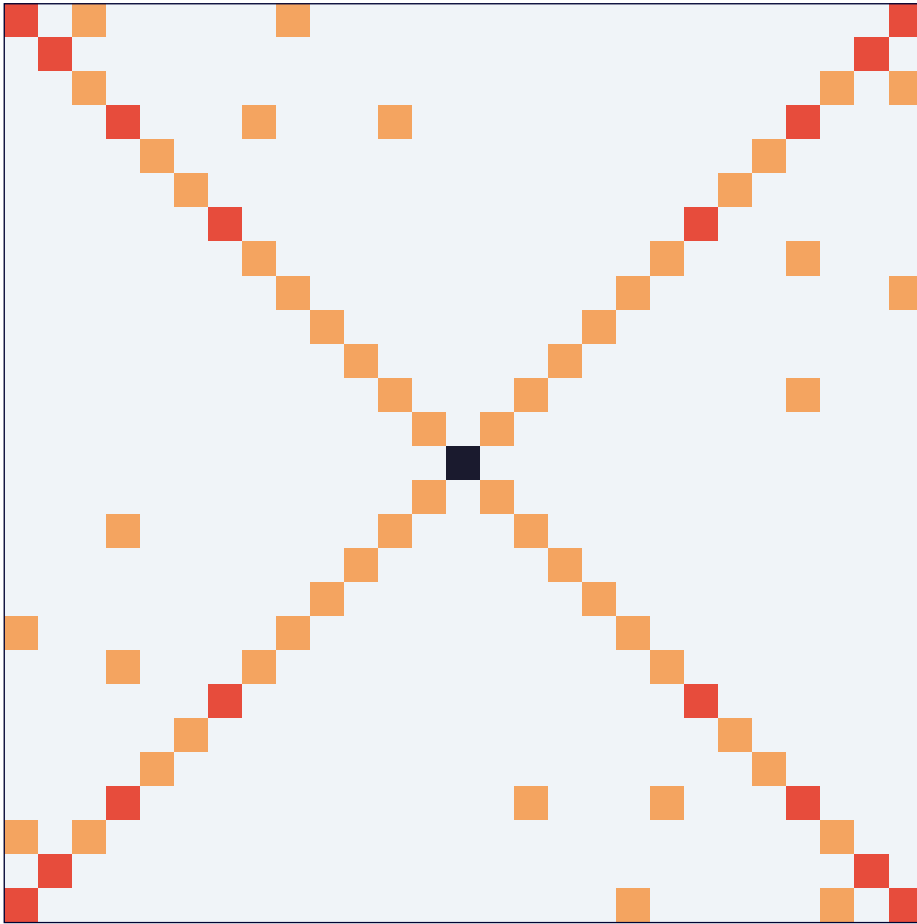
Verified on 831 composites from 4 to 1000: 829/831 match (99.76%). Only exceptions: $N = 8$ and $N = 15$. From $N = 16$ to 1000: **zero exceptions** across 823 composites.

10.3 Visual comparison

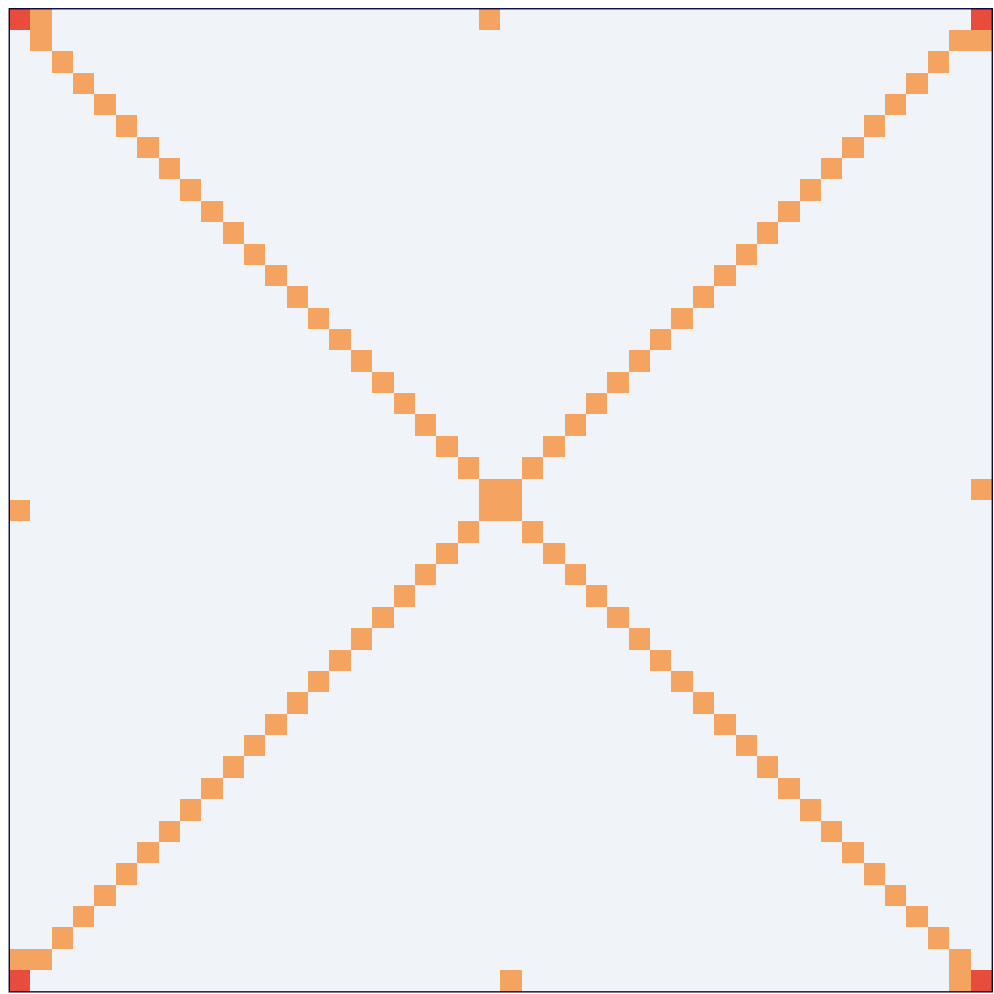
$N = 28$ -- $28 = 29-1 \rightarrow$ Crystalline



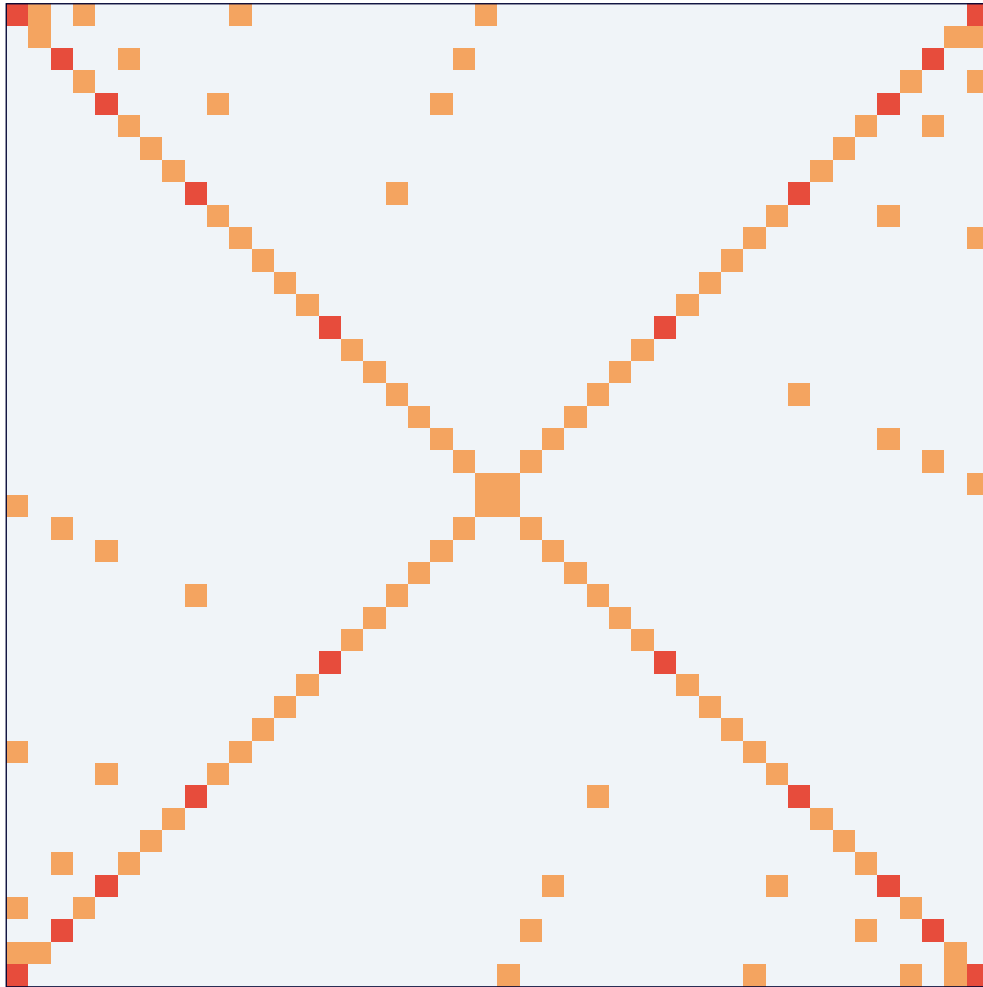
$N = 27 \rightarrow 27 \neq p-1 \rightarrow$ Not crystalline



N = 46 -- 46 = 47-1 -> Crystalline



N = 44 -- 44 != p-1 -> Not crystalline



10.4 Proof: why crystalline = p - 1

The proof connects a new structural observation (reduction to two rotations) with an elementary arithmetic property (numbers with prime sum are coprime).

Only two enemies matter. M and M_{180} have complementary triangles and never collide internally. Only M_{90} and M_{270} can create internal collisions.

Collision condition. A collision between M and M_{90} at (r, c) requires:

$$(r+1) \mid (c+1) \quad \text{AND} \quad (N-r) \mid (c+1)$$

The leaps argument. Row r leaps with step $(r+1)$, the enemy leaps with step $(N-r)$. Their sum is:

$$(r+1) + (N-r) = N + 1$$

If $N+1$ is prime: $(r+1)$ and $(N-r)$ are coprime. Their LCM = $(r+1)(N-r) \geq 2(N-1) > N$. The meeting point is **outside the matrix**. No internal collision. QED

If $N+1$ is not prime: a common factor d exists. LCM drops. Internal collisions appear.

10.5 Corollary: a geometric primality test

To test if p is prime: build the C_4 overlay of $p-1$, check if all collisions are on border and X . Complexity $O(N^2)$ -- no practical value, but conceptually novel: primality determined by the geometry of $p-1$ under rotation, without ever dividing p .

11. Conclusions

Known facts in new clothing. The diagonal theorem (S2) and crystalline = $p-1$ (S10) rest on elementary arithmetic. The contribution is their geometric manifestation and visual discovery.

New quantitative results. $D\% \rightarrow 1/3$ at $O(1/N)$, $P\% \sim \tau(N)/N^2$, Ratio $\sim N \ln(N)$. These are specific to the C_4 representation.

The discovery path. Crystalline Numbers were found visually, matched to $p-1$, then proved via the leaps argument. An example of experimental mathematics.

Open questions:

Formalize border/corner cases. Explain exceptions $N=8, 15$. Extend to D_4 (8 symmetries).

Crystalline numbers -- discovered by looking at colored patterns on a spreadsheet -- turned out to be exactly the numbers preceding a prime. Two leaps with a prime sum never meet inside the field. Mathematics, sometimes, lets itself be seen before it lets itself be proved -- and then lets itself be proved with a metaphor.